Note : This result is not valid for composite numbers.

For Example :

This result cannot be used to find the exponent of 6 in n ! as 6 is composite of 2 and 3.

Illustrating the Concepts :

Find the exponent of 2 *in* 50 ! ?

$$E_2(50!) = \left[\frac{50}{2}\right] + \left[\frac{50}{2^2}\right] + \left[\frac{50}{2^3}\right] + \left[\frac{50}{2^4}\right] + \left[\frac{50}{2^5}\right] = 25 + 12 + 6 + 3 + 1 = 47$$

Illustration - 2 *The number of zeroes in* 100! *are* :

(A) 20 **(B)** 24 **(C)** 97 **(D)** 28

SOLUTION : (B)

We know, $10 = 5 \times 2$

So, to form one 10, we need one 2 and one 5.

Number of 10's will be same as $min\{E_2(100!), E_5(100!)\}$

$$E_2(100!) = \left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] + \left[\frac{100}{2^3}\right] + \left[\frac{100}{2^4}\right] + \left[\frac{100}{2^5}\right]\left[\frac{100}{2^6}\right] = 50 + 25 + 12 + 6 + 3 + 1 = 97$$

$$E_5(100!) = \left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right] = 20 + 4 = 24$$

$$E_{10}(100!) = min\{97, 24\} = 24$$

FUNDAMENTAL PRINCIPLE OF COUNTING — Section - 2

2.1. Addition Principle

If a work can be done in m different ways and another work which is independent of first can be done in n different ways, then either of the two operations can be performed in $(m + n)$ ways.

Illustrating the Concepts :

There are 15 *gates to enter a city from north and* 10 *gates to enter the city from east. In how many ways a person can enter the city ?*

Number of ways to enter the city from north = 15

Number of ways to enter the city from east = 10

A person can enter the city from north or from east.

Hence, the number of ways to enter the city = 15 + 10 = 25

Illustration - 3 *There are 15 students is a class in which 10 are boys and 5 are girls. The class teacher selects either a boy or a girl for monitor of the class. In how many ways the class teacher can make this selection ?*

(A) 5 **(B)** 10 **(C)** 15 **(D)** 150

SOLUTION : (C)

A boy can be selected for the post of monitor in 10 ways.

A girl can be selected for the post of monitor in 5 ways.

Number of ways in which either a boy or a girl can be selected = 10 + 5 = 15

2.2. Multiplication Principle

If one Operation (I) can be done in m different ways and another Operation (II) can be performed in n different ways, then total number of ways in which both of these can be performed together is $m \times n$. If there are more than two operations to be done, then the total number of different ways to do all of them together will be $m \times n \times p \times$ ....

For example :

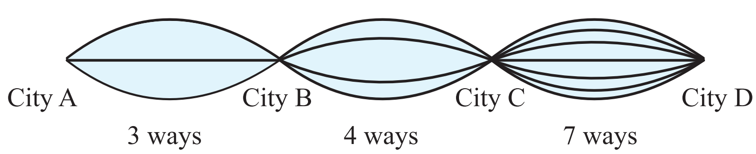
Operation A can be performed in "a, b, c" different ways and Operation B can be performed in p, q different ways, then possible ways to perform Operation A and Operation B together are $a-p$, $a-q$, $b-p$, $b-q$, $c-p$, $c-q$ *i.e.* each way of Operation A is combined by 2 ways of Operation B.

i.e. Total ways = 2 + 2 + 2 = 2 added 3 times = 2 × 3 = 6

In general, total ways = $m \times n$ where m and n are individual ways of performing the operations.

Illustrating the Concepts :

There are 3 routes to travel from city A to City B and 4 routes to travel from City B to City C and 7 routes from C to D. In how many different ways (routes) a man can travel from city A to City D via City B and City C.



The man can perform the task of travelling from City A to City B in ways = 3.

The man can perform task of travelling from City B to City C in ways = 4.

Similarly from City C to City D in ways = 7.

Using fundamental principle of counting,

Total routes to travel from A to D via B and via $C = m \times n \times p$

$= 3 \times 4 \times 7 = 84$ routes.

Illustration - 4 *A city has 12 gates. In how many ways can a person enter the city through one gate and come out through a different gate ?*

(A) 23 **(B)** 144 **(C)** 132 **(D)** 24

SOLUTION : (C)

There are 12 ways to enter into the city. After entering into the city, the man can come out through a different gate in 11 ways.

Hence, by the fundamental principle of counting,

Total number of ways is 12 × 11 = 132 ways.

Illustration - 5 *How many n-digit numbers can be formed using 1, 2, 3, 7, 9 without any repetition of digits when :*

(i) $n = 5$

(A) 120 **(B)** 15 **(C)** 6^5 **(D)** 5^6

(ii) $n = 3$

(A) 12 **(B)** 5^3 **(C)** 60 **(D)** 3^5

SOLUTION : (i).(A) (ii).(C)

(i) 5-digit numbers

Making a 5-digit number is equivalent to filling 5 places.

Places :	□	□	□	□	□
No. of choices :	1	2	3	4	5

The last place (unit's place) can be filled in 5 ways using any of the five given digits.

The ten's place can be filled in 4 ways using any of the remaining 4 digits.

The number of choices for other places can be calculated in the same way.

Total number of ways to fill all five places $= 5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$

Hence, 120 five-digit numbers can be formed.

(ii) 3-digit numbers

Making a three-digit number is equivalent to filling three places (unit's, ten's, hundred's).

Places :	□	□	□
No. of choices :	3	4	5

Total number of ways to fill all the three places $= 5 \times 4 \times 3 = 60$

Hence, 60 three-digit numbers can be formed.

Illustration - 6 *How many 3-letter words can be formed using a, b, c, d, e if :*

(i) *repetition is not allowed*

(A) 60 **(B)** 5^3 **(C)** 3^5 **(D)** 12

(ii) *repetition is allowed*

(A) 60 **(B)** 5^3 **(C)** 3^5 **(D)** 12

SOLUTION : (i).(A) (ii).(B)

(i) Repetition is not allowed :

The number of words that can be formed is equal to the number of ways to fill the three places.

Places :	□	□	□
No. of choices :	5	4	3

First place can be filled in five ways using any of the five letters (a, b, c, d, e).

Similarly, second and third places can be filled using 4 and 3 letters respectively.

$\Rightarrow$ Total number ways to fill = $5 \times 4 \times 3 = 60$

Hence, 60 words can be formed.

(ii) Repetition is allowed :

The number of words that can be formed is equal to the number of ways to fill the three places.

Places :			
No. of choices :	5	5	5

First place can be filled in five ways (a, b, c, d, e).

If repetition is allowed, all the remaining places can be filled in five ways using a, b, c, d, e.

Total number ways to fill = $5 \times 5 \times 5 = 125$.

Hence, 125 words can be formed.

Illustration - 7 *How many four-digit numbers can be formed using the digits* 0, 1, 2, 3, 4, 5 ? (*Repetition is not allowed*)

(A) 5^4 **(B)** 4^5 **(C)** 300 **(D)** 150

SOLUTION : (C)

For a four-digit number, we have to fill four places and 0 cannot appear in the first place (thousand's place).

Places :				
No. of choices :	5	5	4	3

For the first place, there are five choices (1, 2, 3, 4, 5) ; Second place can then be filled in five ways (0 and remaining four digits) ; Third place can be filled in four ways (remaining four digits) ; Fourth place can be filled in three ways (remaining three digits).

Total number of ways to fill $= 5 \times 5 \times 4 \times 3 = 300$

Hence, 300 four-digit numbers can be formed.

Illustration - 8 *In how many ways can six persons be arranged in a row ?*

(A) $6!$ **(B)** 6^6 **(C)** 6^5 **(D)** 5^6

SOLUTION : (A)

Arranging a given set of n different objects is equivalent to filling n places.

So arranging six persons along a row is equivalent to filling 6 places.

Places :						
No. of choices :	6	5	4	3	2	1

Total number of ways to fill all places $= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = 720$

Hence, 720 arrangements are possible.

Illustration - 9 *How many 5-digit odd numbers can be formed using digits* 0, 1, 2, 3, 4, 5 *without repeating digits?*

(A) $4 \times 4\,!$ **(B)** 288 **(C)** 5! **(D)** 300

SOLUTION : (B)

Making a five digit number is equivalent to filling 5 places.

Places :					
No. of choices :	4	4	3	2	3

To make odd numbers, fifth place can be filled by either of 1, 3, 5 *i.e.* 3 ways.

First place can be filled in ways = 4 (excluding 0 and the odd number used to fill fifth place).

Similarly, places second, third and fourth can be filled in 4, 3, 2 ways respectively.

Using fundamental principle of counting, total number of ways to fill 5 places

= Total 5-digit odd numbers that can be formed = $4 \times 4 \times 3 \times 2 \times 3 = 288$ ways.

Illustration - 10 *How many 5-digit numbers divisible by* 2 *can be formed using digits* 0, 1, 2, 3, 4, 5 *without repetition of digits.*

(A) 120 **(B)** 192 **(C)** 312 **(D)** 208

SOLUTION : (C)

To find 5-digit numbers divisible by 2,

We will make 2 cases. In first case, we will find number of numbers divisible by 2 ending with either 2 or 4. In second case, we will find even numbers ending with 0.

Case-I : Even numbers ending with 2 or 4.

Making a five digit number is equivalent to filling 5 places.

Places :					
No. of choices :	4	4	3	2	2

Fifth place can be filled by 2 or 4 i.e. 2 ways.

First place can be filled in 4 ways (excluding 0 and the digit used to fill fifth place)

Similarly, places second, third and fourth can be filled in 4, 3, 2 ways respectively.

Using fundamental principle of counting,

Number of ways to fill 5 places together $= 4 \times 4 \times 3 \times 2 \times 2 = 192$ ways. **. . .(i)**

Case-II : Even numbers ending with 0.

Places :					
No. of choices :	5	4	3	2	1

Making a 5-digit number is equivalent to filling 5 places.

Fifth place is filled by 0, hence can be filled in 1 way.

First place can be filled in 5 ways (Using either of 1, 2, 3, 4, 5).

Similarly places second, third and fourth can be filled in 4, 3, 2 ways respectively.

Using fundamental principle of counting,

Number of ways to fill 5 places together $= 5 \times 4 \times 3 \times 2 \times 1 = 120$ . . .(ii)

Combining (i) and (ii), we get : Total number of 5 digit numbers divisible by 2 = 192 + 120 = 312

Illustration - 11 *How many 5-digit numbers divisible by 4 can be formed using digits* 0, 1, 2, 3, 4, 5*?* (*Repetition not allowed*)

(A) 144 **(B)** 72 **(C)** 288 **(D)** 312

SOLUTION : (A)

Making a five digit number is equivalent to filling 5 places.

A number would be divisible by 4 if the last 2 places are filled by either of 04, 12, 20, 24, 32, 40, 52.

Case-I :

Last 2 places are filled by either of 04, 20, 40.

Fourth and fifth places can be filled in 3 ways of either of 04, 20, 40

Places :	☐	☐	☐	☐	☐
No. of choices :	4	3	2	$\underbrace{\quad\quad}_{3}$	

First place can be filled in 4 ways (excluding the digits used to fill fourth and fifth place).

Similarly, second and third place can be filled in 3 and 2 ways respectively.

Using fundamental principle of counting,

Number of ways to fill 5 places $= 4 \times 3 \times 2 \times 3 = 72$ ways. . . .(i)

Case-II :

Places :	☐	☐	☐	☐	☐
No. of choices :	3	3	2	$\underbrace{\quad\quad}_{4}$	

Last 2 places are filled by either of 12, 24, 32, 52

Fourth and fifth place can be filled in 4 ways (either of 12, 24, 32, 52).

First place can be filled in 3 ways (excluding 0 and the digits used to fill fourth and fifth place)

Similarly, second and third place can be filled in 3 and 2 ways respectively.

Using fundamental principle of counting,

Number ways to fill 5 places $= 3 \times 3 \times 2 \times 4 = 72$ ways. . . .(ii)

Combining (i) and (ii),

Total number of ways to fill 5 places = Total 5-digit numbers divisible by 4 = 72 + 72 = 144

Illustration - 12 *How many six-digit numbers divisible by 25 can be formed using digits* 0, 1, 2, 3, 4, 5*?* (*Repetition not allowed*)

(A) 24 **(B)** 42 **(C)** 256 **(D)** 100

SOLUTION : (B)

Numbers divisible by 25 must end with 25 or 50.

Case-I : Numbers ending with 25

Places :	☐	☐	☐	☐	☐	☐
No. of choices :	3	3	2	1	1	1

Using fundamental principal of counting, total 6 digit numbers divisible by 25 ending with 25

$\Rightarrow$ $3 \times 3! = 18$ numbers are possible.

Case-II : Numbers ending with 50

Places :	☐	☐	☐	☐	☐	☐
No. of choices :	4	3	2	1	1	1

Using fundamental principal of counting, total six digit numbers divisible by 25 ending with 50

$\Rightarrow$ $4! = 24$ numbers are possible.

Hence, total number of multiples of 25 $= 18 + 24 = 42$